Project Description:
Opinion Dynamics and Collective Decisions: Procedures, Behavior, and Systems Dynamics

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1 State of the art and preliminary work

Of democracy we expect that it enables us to utilize collective intelligence such that our collective decisions build and enhance social welfare, and such that we accept their distributive and normative consequences. Collective decisions are produced by voting procedures which aggregate individual preferences or judgments. On the other hand, individual preferences and judgments dynamically form and change by discussion and deliberation. In large groups, these dynamics naturally go beyond the scope of the individual and consequently might show unexpected self-driven macroscopic systems dynamics. This project will explore the interplay of opinion dynamics and collective decision, taking into account the procedural framework, the actual communication and decision behavior of humans as well as the emerging systems dynamics.

Voting procedures intrinsically create obstacles to achieve their democratic aim of properly aggregating people's will. Social choice theory characterizes the possibilities to aggregate a collective decision from individual preferences. Under the condition of non-dictatorship a well-defined aggregation procedure exists only when either it is simply dichotomous (Yes/No or one policy dimension); or if preference heterogeneity is low such that properties of the dichotomous case are resembled; or when some individuals have veto power over all collective decisions (cf. Austen-Smith and Banks 1999). This resembles Arrow's impossibility theorem (1951), and the Gibbard(1973)-Satterthwaite(1975) theorem about the ubiquity of tactical voting, the discursive dilemma (Pettit 2001) and Condorcet's paradox. Majority rule is the only reasonable rule which treats voters and respectively alternatives equally (May 1952). In a more-dimensional policy space the "chaos" theorem (McKelvey 1976) shows that one can steer the collective decision to any point by sequential majority votes. These results make the design of aggregation procedures an intrinsically delicate endeavor.

How individuals form preferences (cf. Druckman and Lupia 2000) before they are put into a procedure of aggregation is left out by social choice theory. The political process we experience in reality is a stream of opinions uttered publicly and privately by a multitude of individuals for a multitude of purposes. For the theory of deliberative democracy (cf. Bohman and Rehg 1997; Mutz 2008) this stream should ideally be a process of transformation of individual preferences through rational discussion towards a consensus about the common good (cf. Mansbridge 1983; Habermas 1998; Elster 1997). Although people are able to deliberate (Steenbergen et al. 2003; Bächtiger et al. 2010), there is evidence that this can moderate (Meffert, Guge, and Lodge 2004) but also polarize views (Wojcieszak 2011). Moreover, depolarization happens only on factual and much less on value-laden issues (Vinokur and Burnstein 1978). In deliberative opinion polling (Fishkin and Luskin 2005) people indeed change opinions and this can lead to a reduction of majority cycles (List et al. 2006) but rarely on salient topics (Farrar et al. 2010). Mansbridge (1983) argues that groups with mostly common interests can benefit from deliberation, while groups with mostly conflicting interests might suffer. Groups might assess this a priori to best choose the way to reach a collective decision, but this would already be a collective assessment confronted with the same obstacles. Finally, also in deliberation there exist incentives for strategic non-truthful utterances undermining its aims (Landa and Meirowitz 2009).

Thus, the voting procedure as well as the dynamic process of deliberation both create obstacles for their democratic aim. Most collective decisions and many political discussions focus on discrete choices as for people or proposals. The underlying judgments and preferences at stake instead are typically of a gradual nature. This holds for budget plans, minimal incomes, tax rates, or assessments of prices, probabilities, and risk, which are numerical values. Also political positions in the spatial model as measured from roll call data (Poole and Rosenthal 1985) and political texts (Slapin and Proksch 2008), as well as personal values and attitudes as measured through surveys are on interval scales. Such **continuous opinions** offer opportunities for adjustment and compromise in direct interaction as well as in a final collective decision that need not be made by coarse-grained majority votes on discrete choices. Further on, pairwise majority votes create majority cycles in budgeting: When three voters are to distribute a fixed amount of money among themselves, then for any status quo allocation, there is a majority of two voters who can propose an allocation where both get more. Averaging preferences might be a smarter option. Interestingly, averaging and similar aggregation procedures are much less studied and rarely used in praxis. Probably this is because it is not clear how selfish, tactical or common-good oriented voters deliberate and vote.

People do not always react rationally or they do it only partly and adapt opinions based on simple heuristics (Gigerenzer and Selten 2001). Further on, in larger groups even rational players can not focus on the opinions and preferences of all other individuals and compute game theoretic equilibria. They have to base their decision on individual interaction, knowing that there are unknown others who form and change the opinion landscape at the same time. Therefore it is important to understand opinion dynamics also on a larger scale, going beyond the scope of the individual. Landscapes of values, preferences and political positions appear fragmented often as long-lasting cleavages (Lipset and Rokkan 1967) or systemic polarization (Pardos-Prado and Dinas 2010). Such opinion landscape can be studied in empirical data from large scale international surveys of consumer ratings from the Internet. Their evolution or the evolution of its stylized facts can be explored by agent-based simulation models where citizens adapt and adjust their opinions through communication (Hegselmann and Krause 2002; Deffuant et al. 2000). This can imply systems dynamics towards consensus, polarization, fragmentation, or extremism, which can be systematically analyzed by methods of the physics of complex socio-economic systems (cf. Helbing 1995; Schweitzer 1997; Fortunato and Castellano 2007).

Nowadays, modern communication technology scaled and sped up the direct and decentralized deliberation of citizens to the societal level. This will possibly continue. Further on, electronic voting makes it possible to also extend collective decision to the societal level. To properly cope with these developments and to properly use their opportunities we need to better understand opinion dynamics and collective decision based on a larger scale. The project will contribute to this by integrating the understanding of

- the intrinsic limitations of procedure of aggregating continuous preferences,
- the process of discussion and decision about numerical values through the rational-choice perspective,
- the interwoven bargaining, truth-finding, and coordination of individuals and groups through experiments,
- the emerging systems dynamics created through the interaction of individuals in large groups, and
- the stylized facts of existing data on continuous opinion landscapes.

The long term goal of this integrated approach is to contribute to the development of direct democratic innovations (Smith 2009) of deliberation and aggregation for tasks such as collective assessment, redistributive taxation and participatory budgeting, such that democracy can elicit and construct the wisdom of the crowd (Surowiecki 2004) rather than the madness of the crowd (Mackay 1848).

The following six sections further point out the state of the art from six scientific perspectives. The modeling framework shown there serves as preliminary work for the execution of the projects' work program (Sec. 2.3) where each work package relies on cycling through some of the perspectives (cf. Fig. 5). In particular, the following sections lay out the normative routes for designing aggregation procedures based on social choice theory, show the spatial modeling framework, outline how preferences and behavior will be modeled, and show how monetarily incentivized experiments, agent-based simulation and data analysis will be used.

Normative Intention

1.1 The trilemma of democracy and the role of opinion dynamics

The paradoxes, impossibility results and dilemmas of social choice theory are rephrased by the *democratic trilemma* (List 2011). Any aggregation rule can only fulfill two of three principles: robustness to pluralism (or a universal domain), basic majoritarianism (nothing shall be implemented when a majority prefers something else), and collective rationality (aggregated preferences are transitive and complete). List (2011) lays out a *logical space* of procedures of democratic preference aggregation by relaxing each of the three principles.

Relaxing collective rationality by allowing intransitivity of collective preferences would weaken accountability. Relaxing the condition of completeness results in a rule that delivers “no decision” under some conditions. A non-decision might sometimes be appropriate, but on the one hand budgeting and taxation are examples where a decision is necessary to start coordinated societal action and on the other hand no decision is always a decision for the status quo.

Relaxing basic majoritarianism means that a collective decision can be made by a procedure, even when there is another possible decision which would win against it by a majority vote. One procedure is a terminating chain of dichotomous majority votes, where the decision about the proposals or about the order of votes is externalized to another entity (the agenda-setter) or a different procedure. Krause (1995) showed that an essentially lexicographic order of importance in policy dimensions is natural in multi-criteria decision making. This does not seem appropriate for all kinds of decisions, e.g. not for budgeting when there is no seniority of budget items. An endogenous way of relaxing majoritarianism is to find a suitable and acceptable procedure of aggregation which defines a “best fitting” collective outcome. Considering the distribution of money among three projects, a possible procedure of aggregation would be that each agent hands in a proposal and the average of all three proposals is implemented. Lehrer and Wagner (1981) showed that weighted arithmetic averaging is the only procedure which fulfills the conditions of zero unanimity and neutrality of projects. A practical implementation of a collective decision procedure of that kind is *participatory budgeting* as implemented in 1992 in Porto Allegre (Santos 1998). The aggregation procedure is similar to a Borda count and aggregated results are directly transfered into budget sizes. The idea of participatory budgeting spread over the world (Sintomer, C. Herzberg, and Röcke 2008). Such “direct” aggregation procedures without votes over proposals, e.g. based on averaging, appear particularly suited for decisions about numerical values.

Relaxing robustness to pluralism puts the burden back on voters to deliver individual preferences for which aggregation can stand majority votes, and which delivers a collectively rational decision, e.g. by fulfilling conditions of value restrictions (possibility result of Sen 1966). This is most easily fulfilled when individuals have a consensus about collective preferences, which is the ideal aim of deliberative democracy. Dryzek and List (2003) argue that deliberation – if not able to produce consensus – can still promote achievement of a *metaconsensus*, where individuals agree such that alternatives form an ordered set (as the political left—right continuum) from which the electorate can then stably decide on for the median voter’s choice (Hotelling 1929; Black 1948; Downs 1957). This can be generalized to alternatives which form a partially ordered set (a lattice) (Pivato 2009). In simultaneous decisions about more than one numerical value a metaconsensus would be a collective agreement about a (partial) order of n-dimensional space, e.g. a projection to an appropriate line. Individuals might achieve this by entering a pre-decision deliberation on collective beliefs and collective preferences.

We will follow two routes out of the democratic trilemma for collective decisions about numerical values:

1. Procedures of direct aggregation of numerical values which are acceptable but do not standing basic majoritarianism.
2. Understand which procedures and hidden dynamics of deliberation foster conditions which help to achieve consensus or at least metaconsensus.

Model Procedure

1.2 Modeling framework for continuous opinion dynamics and collective decision

To combine the different aspects of continuous opinion dynamics and collective decision a common modeling framework is needed which is able to represent real-world data generating processes, to capture rational choice models, and dynamical agent-based models. Fig. 1 shows this framework. It is composed of two parts: the opinion dynamics part modeled as a stream of uttered opinions (⇛) and the collective decision part (↓) modeled as an aggregation function mapping individual decisions to a collective decision. Both parts are structurally independent but linked through the fact that the same players discuss about the same numerical issue on which they later decide (⇛↓).

A common definition of an **opinion** is: A judgment, viewpoint, or statement about matters commonly considered to be uncertain or subjective, and thus a result of guessing, interpretation or emotion. We use “opinion” as an umbrella term. An example from social psychology is an **attitude** (cf. Heider 1946; Anderson 1971) as for example an evaluation of a consumer product. Another example for opinions are self-assessments as for example used in surveys to measure **values** or other **preferences**. Also included under the umbrella of the notion opinion are proposals for a collective decision, such as a desired tax rate or a budget plan. In the models used in the project an opinion usually boils down to numbers or a vector of numbers from a continuum or a quasi-continuum, as a Likert or a rating scale. Opinions are held and uttered by **Individuals**. Based on the scientific context the individual is also called **agent** (agent-based modeling and simulation), **subject** (experiments), **player** (game-theory), **voter** (person handing in a ballot to a voting procedure), or **citizen** (eligible member of a political community).

Modeling continuous opinion dynamics

Variables and data The **stream of opinions** is a collection of triples $(i, t, x) \in N \times \mathbb{T} \times \mathbb{O}$ based on the set of **individuals** $N = \{1, \dots, n\}$, the set of **time steps** $\mathbb{T}$ (the reals or the integers), and the set of possible **opinions** $\mathbb{O}$. It represents that agent i has opinion x at time t. An opinion about k numerical values is specified

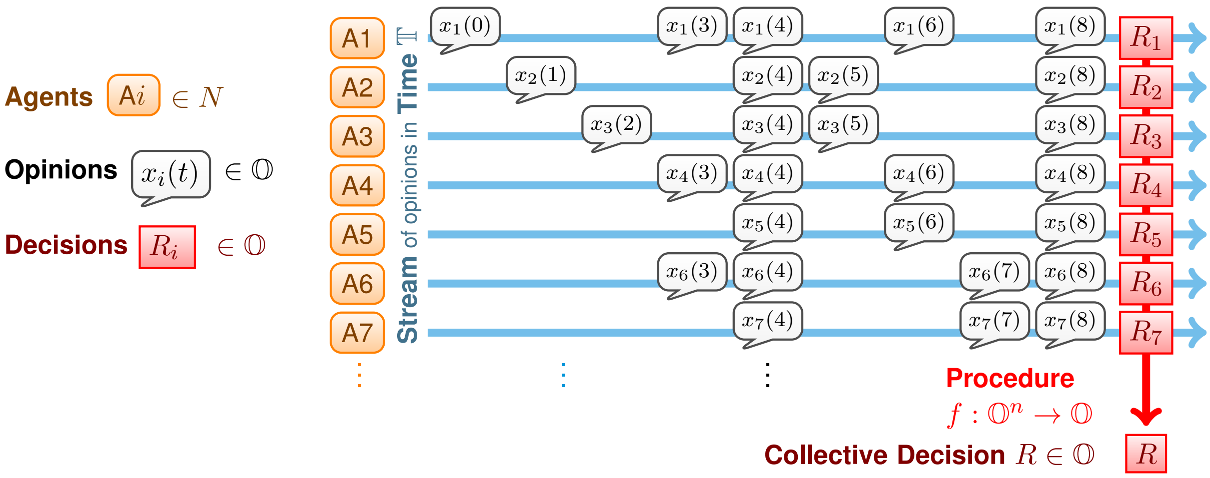


Figure 1: **Modeling framework** for continuous opinion dynamics and collective decision. The example displayed by $x_i(t)$'s can be read as: In ordered speeches, A1, A2 and A3 utter their opinion. Then A1, A4 and A6 give a statement together. This triggers a straw vote. Subsequently, three bi- and trilateral meetings lead to new statements. After a final straw vote, anonymous votes are collected for a decision.

by a **continuous opinion space** as vectors of real numbers $x \in \mathbb{O} = \mathbb{R}^k$. A real-world example for a stream of opinions is the question about the ideological left—right self assessment in panel surveys like the German Socio-Economic Panel (SOEP). Another example is a stream of $1\bigstar$—$10\bigstar$ online consumer ratings, e.g. for movies. The opinion space resembles the policy spaces in the spatial model of politics (Downs 1957; McKelvey 1976). As no agent utters two opinions at the same time, the pair (i, t) can serve as a unique identifier in a data table.

Agent-based stochastic dynamical system With the rule that no one can have two opinions at the same time, the stream can be modeled as an n-agent (stochastic) dynamical system in the state space $\mathbb{O}^N$ with a state vector $x(t) \in \mathbb{O}^N$ where $x_i(t) \in \mathbb{O}$ denotes the opinion of agent i at time t. To specify the model in discrete time we need to define a (stochastic) function that computes $x(t+1)$ from $x(t)$ or possibly also from former time steps. This boils down to the behavioral definition of how agent i with current opinion $x_i(t)$ comes to a new opinion when confronted with the opinions of others in the opinion profile $x(t)$ and possibly also earlier time-steps and other external parameters.

Game of opinion formation Game theory instead does not start with behavioral assumptions about how to form opinions given the available information but with the definition of preferences in the form of utility functions on which basis players decide for their actions forward looking. They rationally take into account that also others are forward looking. When players have utility assigned to the opinion profile of all individuals at time t they play a *Bayesian game with communication*. This game neglects the part of the collective decision. The game consists of a classical Bayesian game and a communication system (cf. Myerson 1991, Sec. 6.3).

The Bayesian game is defined by a set of actions ($\mathbb{O}$ for all players), types (representing players' private information), beliefs (individual probability distributions over the types of others) and utility functions for each agent. Agent i's utility function maps from the set of all players' actions to a payoff $(u_i)_{i\in N}(x_1, \dots, x_n)$ mapping the set of actions to payoffs.

In a general *communication system* players can send and receive messages from other players which are costless, nonbinding and nonverifiable. The intention of such "cheap talk" (Farrell and Rabin 1996)) is to change the beliefs of others (cf. Austen-Smith 1992). A simple example of a communication system is a system where players can repeatedly send opinions from the opinion set $\mathbb{O}$ to the attention of other players. With this they might express their opinion about the state of the world (*"I think $x \in \mathbb{O}$ is correct."*), their own type (*"My preference is x."*), or an appeal for a collective opinion (*"Let us say x."*).

Modeling collective decision

Aggregation procedure Following Austen-Smith and Banks (1999) an aggregation rule f maps the preferences of all voters to a collective preference. In its most general form, a preference is a relation $\mathcal{R}$ over the set of outcomes which are collected in a preference profile $\rho = (R_1, \dots, R_n) \in \mathcal{R}^n$. In our case, the set of outcomes is the opinion space $\mathcal{R} = \mathbb{O}$. A practical aggregation rule for most general preferences has thus to take the full utility function of each player as an input. Utility functions in a spatial model as ours are typically assumed to be **single-peaked** (Black 1948) at an **ideal point** with a unique global maximum and with individual's utility decreasing with distance from this ideal point. Under the assumption that individuals have similar single-peaked preferences for the collective decision but with different ideal points it seems reasonable to reduce each voter's input and the collective decision to one value from $\mathbb{O}$. Thus, an aggregation rule in our project maps a set of numbers to one number. The same applies for vectors in multidimensional opinion spaces.

In the project, we will use the three types of aggregation rules listed in Tab. 1. The three aggregation rules coincide with the three standard measures of central tendency of a statistical distribution: mode, median, and mean. They have an increasing minimal level of measurement: nominal, ordinal, and interval scale. The definition of corresponding rules in more dimensional opinion spaces is left for the work in the project.

Median The median is the only rule which stands basic majoritarianism. By definition, there is no other opinion on the real line which a majority prefers against the median. This is why Galton (1907) proposed it as the democratic rule for aggregation of judgments. Bassett Jr. and Persky (1999) further support it as a voting method which is robust against outliers, based on the analogy to robust statistics. Giving this good property it is surprising that the median has rarely been proposed or implemented as a direct procedure of decision, but is mostly studied in the light of the median voter theorem (Downs 1957). An example is an experiment for redistributive decisions about a tax rate by Esarey, Salmon, and Barrilleaux (2012). One should mention a weakness of the median: It neglects the opportunities for intermediate compromising decisions, especially in highly bipolarized situations, say when about half of the population prefers 0% tax rate while the other half prefers 100%.

Mean The arithmetic mean of voters' values can aggregate compromising solutions even in extremely polarized situations, but the mean usually does not stand all majoritarity votes. Real-world examples of collective decisions by means are averaging of judges in sports competitions such as gymnastics and the procedures to compute daily interbank offered interest rates such as Libor or Euribor, where banks from a panel submit their preferred rates. In fact, procedures often use a trimmed mean where the top and bottom 23% or 15% of the distribution are neglected before averaging. Raising that percentage to 50% drives the trimmed mean to be the median.

Mode is the value which is most often voted for. Using it resembles voting for discrete choices by a simple first-past-the-post voting. Some further procedural specifications for handling ties are necessary as further voting rounds or random draws. Of course, with a continuum of choices some pre-decision communication seems obligatory such that players do not need to rely on focal points. Minimal quotas for decisions such as a majority or two-thirds could be introduced for the sake of allowing "no decision" as an outcome.

Table 1: **Aggregation rules** for numerical values in one dimension $\mathbb{O} = \mathbb{R}$. Arguments in the text assume that voters have single-peaked preferences and vote for their ideal point.

Game of collective decision The game of opinion formation defined before did not include the aggregation of a collective decision. By adding a procedure of collective decision f, individuals can assign utility to the collective decision R instead of the whole opinion profile $x(t)$. Then they face a *Bayesian collective-choice problem* based on utility functions $u_i(R)$, which are derived from all individual decisions $u_i(f(R_1, \dots, R_n))$ (cf. Myerson 1991, Sec. 6.4). The game perspective takes into account that voters might not vote sincerely by voting for their ideal point, but strategic to maximize the collective decision to lie as close to their ideal point as possible. The opinions uttered beforehand can additionally be interpreted as intended action *"I will vote for x."*.

The strength of this dual view as a dynamical system and as a game is that it captures both forward and backward looking individuals. Both aspects are valid, because opinions have both, a positivist evaluative aspect based on the past as well as normative and strategic aspect looking at the future. An opinion is often a mix of and evaluation and interpretations of the available opinion information as well as a normative statement and a strategic proposal to shape collective preferences facing an upcoming collective decision. A strategic game might change through the non-strategic process of evaluation and interpretation (e.g. through change of salience among dimensions, cf. Dietrich and List (2011)). On the other hand, even if players utter opinions only based on simple evaluation heuristics they might be considered as strategic messages by other players.

The next section focuses the game-theoretic aspect of games of opinion formation and games of collective decision by specifying the preferences which will be used in the project.

Rationality Heuristics

1.3 Rational choice modeling of cheap talk in multi-agent bargaining and coordination

Theorists of deliberative democracy acknowledge that in political conflicts informational and distributive aspects coexist (Landwehr 2010), and thus politics is a market for bargaining as well as a forum for the exchange of information and the coordination of collective action (Elster 1997). To model this, we acknowledge that individuals have individual, epistemic and social preferences which triggers a behavioral mixture of bargaining, truth-finding, and coordination. The conceptions resemble *individual choice*, *social planner*, and *social preferences* in Traub, Seidl, and U. Schmidt (2009). They are related to the main theories of voting behavior: voting for the party closest to your position (Downs 1957); voting for the party who you truly identify with (Michigan model, Campbell et al. 1960), or voting for the party opinion leaders in your social network vote for (multistep flow theory, Lazarsfeld, Berelson, and Gaudet 1944). Modeling in the project assumes that individuals possess all three preferences but weigh and use them in diverse ways with respect to the problem and their heterogeneous personalities. The three types of preferences are described in Tab. 2. Their specification, analysis and experimental operationalization is subject of the work packages WP1 and WP2.

Individual preferences are based on ideal points in the opinion space which are usually private information. In a game of opinion formation players with purely individual preferences $u_i^{\text{ind}}(x_i)$ will simply chose the ideal point as opinion. Thus, maximization is trivial when individual preferences are not combined with other preferences. In a game of collective decision in contrast players with purely individual preferences $u_i^{\text{ind}}(R)$ want the collective decision to be at their ideal point. As the collective decision aggregates all individual decision by rules as mean, median or mode (cf. Tab. 1) players might decide for something else then their ideal point, when they have beliefs about the decisions of others.

Epistemic preferences are based on objective measures. There are two typical conceptions of epistemic preferences: social welfare and truth. Social welfare needs to be based on a common definition, e.g. as sum or minimum of individual payoffs regarding the collective decision. The truth is what individuals agree on to be objectively correct, though it might be unknown or not yet known. Thus, utility can only be assessed on the basis of beliefs at present time. The truth of a value in a continuum can also be assessed gradually, e.g. by a single-peaked utility function peaking at the unknown correct value. Social welfare can be derived from truth by looking at its consequences for individual payoffs. For example, the correctness of an estimate about the rise in sea levels or next year's economic growth can make a difference in welfare when policies are based on it. Because of the assumption of objectivity the utility function is unique for all players in games of opinion formation and collective decision: $u^{\text{epi}}(x_i(t))$, $u^{\text{epi}}(R)$

Social preferences appear when players care also about the preferences (or payoffs, or opinions) of others in their utility function (Fehr and K. M. Schmidt 1999; Charness and Rabin 2002). Social preferences make a game of opinion formation an interesting game. Social utility functions regarding the opinion profile $u_i^{\text{soc}}(x_1(t),\dots,x_n(t))$ can model games of coordination of opinions, where players are interested to agree with the opinions of others. In contrast, social preferences about individualization peak when one has an opinion which nobody else shares. Also in a game of collective decision social preferences might play a role, when players are aware of the individual payoffs others receive from the collective decision $u_i^{\text{soc}}(R) = g(u_1^{\text{ind}}(R),\dots,u_n^{\text{ind}}(R))$. This can incorporate that differences in payoff shall be small, or that everybody gets a certain minimum.

Table 2: **Preferences** in games of opinion formation and games of collective decision.

Multi-player bargaining about *individual preferences* is a typical subject to cooperative game theory about coalition formation. The assumption of transferable utility models that coalitions can share the individually gained payoff equally after achieving it. In decisions about values or policies this assumptions need not be valid. Modeling in the project shall study multiplayer bargaining without transferable utility.

An example of *social preferences* is the coordination problem about a societal convention. It can be modeled by a utility function which peaks when all players have the same opinion, making every consensus a Nash equilibrium. The selection of one equilibrium is one of the most interesting questions of game-theory (Huyck, Battalio, and Beil 1991) which is often solved by focal negotiation (cf. Schelling 1957). A social norm has been defined as a convention which solves a cooperation problem (Ullmann-Margalit 1977). For example, the classical public good game can be solved with everybody cooperating when a large enough share of utility derived from a convention is included in everybody's utility function. The experiments of the project shall contribute to an understanding of equilibrium choice by exploring how individuals approach an equilibrium through communication.

In the opinion dynamics stage before the utterance of the final opinion or final decision, all opinion utterances are *cheap talk*, which enables players to reveal their private information. This is beneficial only if agents share at least some common interest (Austen-Smith 1992; Forges 1990). This makes the game a *game with communication* (cf. Myerson 1991, Meirowitz 2007 for the spatial model of politics). In game-theory the use of communication is often reduced to *mechanisms*, where players report their private information confidentially to a mediator, who confidentially recommends decisions for every player. This is because the revelation principle delivers that for any equilibrium of an arbitrary communication game, there exists an *incentive-compatible* mechanism where agents have no incentive to lie about their private information and no incentive to not take the recommended opinion, but with mechanism of mediation the choice of equilibrium lies purely with the mediator, which is a problem.

An example similar to our framework is the jury game (Austen-Smith and Banks 1996). A jury facing a guilty/innocent decision might decide falsely positive more likely under consensual quorums than under majority rule. Gerardi and Yariv (2007) instead show that communication makes truth-telling an equilibrium. It is subject to the project to extend these results about dichotomous choices to choices in a continuum.

The *epistemic* aim of continuous opinion dynamics and collective decision is to reach good collective assessments (Dietrich 2010). Aumann (1976) has shown that players who agree that their observations come from a common prior can not have different posterior assessments about probabilities when these are common knowledge. McKelvey and Page (1986) showed how this impossibility of agreeing to disagree leads to convergence of individual beliefs. A way to reach consensus about continuous assessments is that individuals repeatedly revise their beliefs by weighted means of the reported beliefs of others as in a Delphi process (Rowe and Wright 1999). DeGroot (1974) showed conditions under which a consensus will be reached. This triggered a mathematical literature about the conditions for convergence to consensus (see citations in J. Lorenz and D. A. Lorenz 2010), which recently reached economics again (DeMarzo, Vayanos, and Zwiebel 2003; Golub and Jackson 2010; Buechel, Hellmann, and Klößner 2012; Golub and Jackson 2012; Eger 2013) with Acemoglu and Ozdaglar (2011) acknowledging that these models are still in their infancy. The project will experimentally explore how updating via opinion dynamics happens in the interplay with individual and social preferences.

Opinion dynamics is about subjective issues. Therefore, another typical assumption of game-theoretic models is often not fulfilled: a common prior is missing and players have to live with inconsistent beliefs. This may lead for example to rational belief polarization (Dixit and Weibull 2007; Jern, Chang, and Kemp 2009) which undermines consensus. Spector (2000) instead showed that in a more-dimensional setting conflict might become at least one-dimensional through rational debate. Both phenomena will be studied experimentally in the project.

Procedures of direct aggregation as listed in Tab. 1 appear natural for decisions about numerical values, but each of the rules might trigger some strategic adjustment behavior. For examples, a player can manipulate the collective decision to any value under the mean in unbounded opinion spaces once it knows all other values. Even a player with epistemic preferences might hand in more extreme estimates to "pull" the mean to the right direction, similar to the directional model of voting (Merrill and Grofman 1999) where a voter votes for a more extreme candidate in the expectation that she can pull the collective policy only half way towards its position. The recent Libor scandal about fixation of interest rates in the interbank market is another instance of non-truthful reporting under an averaging procedure of aggregation (Eisl, Jankowitsch, and Subrahmanyam 2013). In the project, aggregation procedures from Tab. 1 will be analyzed in their interplay with the preferences from Tab. 2. They will be extended to multidimensional decision problems, and tested for certain policy decisions, acknowledging that players do not only have epistemic but also individual and social preferences and not necessarily work with consistent beliefs based on a common prior and without transferable utility.

Empirics
Microlevel

1.4 Opinion dynamics experiments and human behavior

How players and groups really play games of opinion formation and collective decision and how the individual, epistemic, social preferences, and procedures interplay is an empirical question which the project explores and tests with monetarily incentivized group decision experiments (WP2). Monetary incentives impose preferences in subjects. The experimental setup in a computer laboratory enables to control communication and the procedures of decision. The method leads to high internal validity of results and is increasingly used also in political science (Kittel, Luhan, and Morton 2012).

For example, convergence to the median voter has been shown even when ideal points of voters and the positions of candidates are both private information (Collier et al. 1987). McCubbins and Rodriguez (2006) tested the effect of costly and free deliberation for solving math problems. They found that deliberation can also decrease social welfare and that it increases welfare when it functions as an expertise system. In the dictator game it has been shown that group decisions can be less altruistic under chat communication while they are more altruistic in face to face communication (Luhan, Kocher, and Sutter 2009). In an experiment about honest reporting of a die roll Fischbacher and Heusi (2008) showed that people tend to lie to get more payoff but often they compromise between the truth and the maximal payoff. Based on this finding Diekmann, Przepiorka, and Rauhut (2011) showed how an aggregate feedback on this norm violation has a contagious effect. Majority decisions on redistribution based on income from real effort (Cabrales, Nagel, and Rodríguez Mora 2012) showed very robustly that the rich are against, while the poor are in favor. Goeree and Yariv (2011) showed that deliberation in the jury game indeed makes different quorum rules more equal.

The latter two examples forced groups to make dichotomous decisions. Experimental operationalizations with gradual collective decisions and controlled communication as in our framework are much less common. Exceptions are the recent works of Kittel, Paetzel, and Traub (2012), J. Lorenz, Rauhut, Schweitzer, et al. (2011), and J. Lorenz, Rauhut, and Kittel (2012) which build the project's basis for experimental operationalization. An example is shown in Fig. 2.

The experiments to be designed in work package WP2 will be build on a mixture of private individual preferences with heterogeneous ideal points and epistemic preferences coupled either by additional induced social preferences in games of opinion formation or by a collective decision through an aggregation rule (see Tabs. 2 and 1 for details on preferences and aggregation rules). Typically, these games will have multiple and often a continuum of equilibria even in pure strategies similar to pure coordination games. Therefore, the empirical question will be if an equilibrium will be reached and which. Focal points (Mehta, Starmer, and Sugden 1994) and payoff dominance have been shown to be typical criteria (Huyck, Battalio, and Beil 1991; Feri, Irlenbusch, and Sutter 2010). The equilibria in our games will typically have no payoff domination as well as no transferable utility such that the typical framework of cooperative game theory does not apply. Krupka and Weber (2013) presented an experiment of norm coordination in the dictator game which touches our experimental framework of mixing individual preferences with the need to coordinate but not in a fully continuous action space and without communication.

It is known from physics that measurement on the atomic level is not objective as it modifies reality and makes other measurements impossible. The same should hold for humans as atomic unit in experiments. Further on, it often turns out that rational choice models predict human behavior only for certain payoff specifications (Goeree and Holt 2001). Therefore, design of experiments in the project will not solely focus on model validation, model fitting and model extension, but also on exploration and innovation for real world application.

Psychology has also studied experimentally how individuals and groups decide. In contrast to the standard assumption of rational choice, it has been shown in cognitive psychology that individuals regularly choose stochastically dominated alternatives (Diederich and Busemeyer 1999). This suggests that an individual's dichotomous decision can be well described by a stochastic process of an underlying continuous valence hitting a threshold. Repeated estimation tasks and subsequent analysis of the wisdom-of-crowd effect showed that knowledge might be probabilistically represented in the brain (Vul and Pashler 2008; Rauhut and J. Lorenz 2010). Nonrational theories of how decision making (Gigerenzer 2001) as simplifying heuristics or bounded rationality (Gigerenzer and Selten 2001) have shown to describe successful human behavior well.

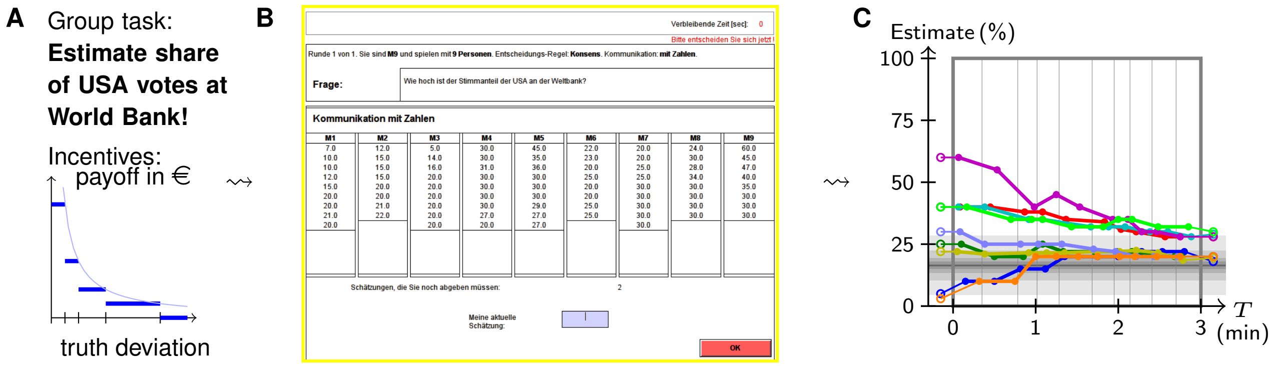


Figure 2: Example experiment (from J. Lorenz, Rauhut, and Kittel 2012). **A** Incentivized estimation task for a committee, **B** operationalization of deliberation with numbers, **C** trajectories of opinions as experimental data.

Social psychology has delivered theories how individuals adjust their judgements, attitudes, and values with respect to others, leading for examples to effects such as group polarization (Myers and Lamm 1976) or consensus (Davis 1996). Some of these behavioral rules can be derived from the principle of dissonance reduction (cf. Festinger 1954; Groeber, J. Lorenz, and Schweitzer 2013).

Even further away from rationality, evolutionary game theory can explain the emergence and coexistence of different norms and behavior regarding collective action (Ostrom 2000) or the emergence of a sanctioning institution (Sigmund et al. 2010) by reinforcement through higher reproduction. Similar dynamics are implied by cultural imitation or backward-looking learning (Macy and Flache 2002) based on copying successful behavior from peers or past explorations (Traulsen et al. 2009). For values, Boehnke (2001) analyzed their transmission from parents to children compared to value change through societal context.

Thus, individual behavior can rely on embeddedness in social context and group behavior can rely on dynamics which goes beyond rationality and beyond the scopes of the actors (Macy and Flache 1995). To study such systemic dynamics, an accompanying analysis by agent-based models and computer simulation is useful for the project (WP4). To that end, a list of possible human behavior in opinion dynamics from psychological models is listed in Tab. 3 which serve as building blocks for agent-based modeling as outlined in the next section.

Utterance is to speak out the own private opinion. This is thought to happen only when some internal or societal parameter hits an activation threshold (cf. Diederich and Busemeyer 1999; Granovetter 1978).

Homophily is the tendency to interact only with similar individuals (McPherson, Smith-Lovin, and Cook 2001; Feld and Grofman 2009).

Adoption is to take over the opinion of someone else, for example for conformity's sake (cf. Asch 1955)).

Balancing is to change to an opinion which leads to a situation which is individually perceived as balanced (cf. Heider 1946). An example is to switch to a median opinion within social context.

Integration is the change through the integration of an opinion from the social context's stimuli (cf. Anderson 1971), typically by various options for (weighted) averaging (cf. Hegselmann and Krause 2005).

Individualization is to take an opinion which is different from others' (Mäs, Flache, and Helbing 2010).

Table 3: **Human behavior** in opinion dynamics.

Systems Dynamics

1.5 Agent-based simulation and the physics of opinion dynamics

Agent-based simulation of artificial societies has been used in political science (see Epstein and Axtell 1996; J. Lorenz 2012b) and in particular in the spatial model (Laver 2005) where parties use heuristics to maximize votes from nonstrategic voters with fixed preferences. What is not modeled is a dynamical behavior of voters adjusting their preference, but only their party choice. Agent-based modeling in the project might be seen as taking place in the same policy world as Laver's model but with voters which adjust preferences dynamically while parties are omitted.

One suitable model for this is the bounded confidence model (Krause 2000; Hegselmann and Krause 2002;

Deffuant et al. 2000; Weisbuch et al. 2002), analyzing the evolution of opinion clusters. The model triggered a large stream of literature (see J. Lorenz 2007). Many variations of the model are summarized in an online simulation programmed in Netlogo (J. Lorenz 2012a).

The aim of agent-based simulation is to understand driving forces of opinion dynamics which are hidden in the interaction of individuals. Physics studies particle systems and how their local interaction can lead to tipping phenomena like phase transitions at critical temperatures, as sudden magnetization or condensation. Sudden changes in opinion landscapes may be described similarly as physics of complex socio-economic systems (Helbing 1995; Schweitzer 2003; Castellano, Fortunato, and Loreto 2009). Physical models often use temperature-dependent stochasticity where temperature plays the same role as the parameter quantifying beliefs about the error of selection of strategies in the quantal response equilibrium (McKelvey and Palfrey 1995).

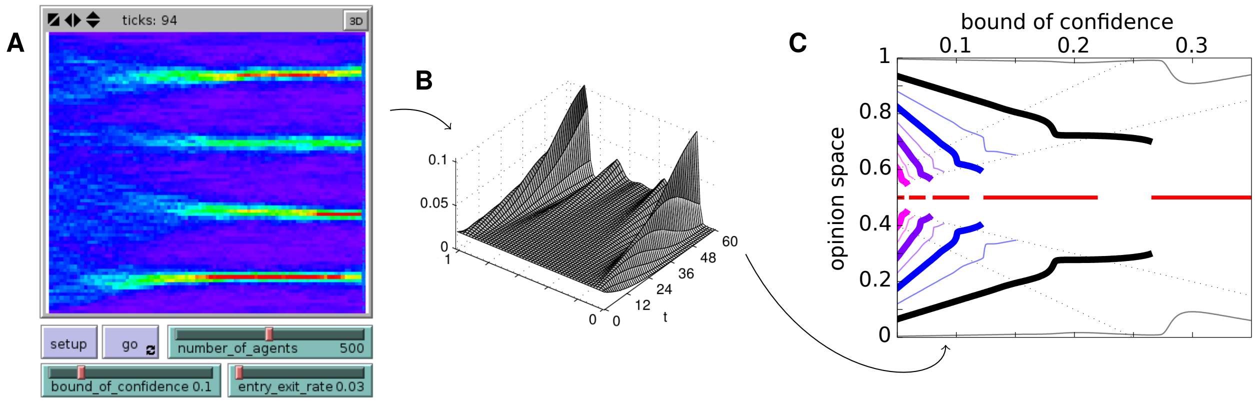


Figure 3: **A** Agent-based simulation of a bounded confidence model (adapted from J. Lorenz 2012a), **B** density-based computation of the opinion landscape's evolution, **C** bifurcation diagram of the stabilized cluster patterns.

Every agent-based model has a density-based description. The equations modeling change of opinion transform to equations for the change of the opinion density in the opinion space (Ben-Naim, Krapivsky, and Redner 2003; J. Lorenz 2007). Fig. 3B shows an example. Density-based equations describe the evolution of opinion landscapes instead of individual trajectories, capturing the behavior of the system in the limit of many agents. These descriptions allow numerical computation of the evolution, which gives a good overview on the stable and attractive opinion landscapes. Stable opinion landscapes under the bounded confidence assumption are characterized by opinion clusters, i.e. small regions in the opinion space where a large fraction of agents concentrates. For high confidence a consensual central cluster evolves. For lower confidence two opposing clusters evolve. Number and location of clusters are plotted against the confidence bound in a bifurcation diagram (Fig. 3C). When some regular influx of new opinions is added (cf. Pineda, Toral, and Hernandez-Garcia 2009; Pineda, Toral, and Hernandez-Garcia 2011) this can lead to abrupt changes between states with different number of clusters and to abrupt changes of the median voter's position similar to switching of majority in many party systems (testable in Netlogo J. Lorenz 2012a).

In work package WP4 new agent-based models and density-based descriptions will be developed based on J. Lorenz (2012a) by taking the behavioral building blocks from social psychology listed in Tab. 3 into account.

Empirics
Macrolevel

1.6 Opinion landscapes in politics and society

An alternative way to develop agent-based models is to start with empirical observation of opinion landscapes. Instead of defining local interactions ad hoc from psychological theory or experimental observation one tries to deduct them from the macroscopic observation. The idea is to find a (stochastic) processes that delivers similar distributions of opinions (cf. Macy and Willer 2002). This needs to be based on a proper description and characterization of opinion landscapes which goes beyond a characterization by mean and standard deviation. They can also lead to adjustments or invention of new social indicators (Delhey and Kohler 2011). International and national surveys are full of gradual questions which deliver opinion landscapes which are close to continuous, e.g. World/European Value Survey (WVS/EVW), European Social Survey (ESS), European Quality of Life Survey (EQLS), German Socio Economic Panel (SOEP).

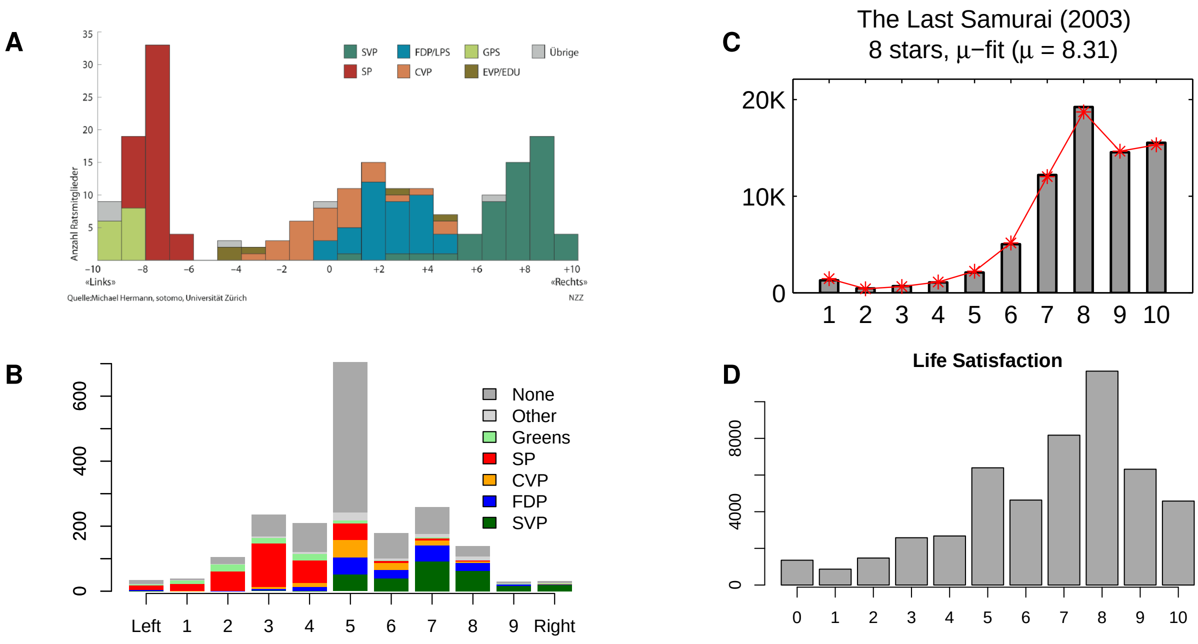


Figure 4: Triple-peaked opinion landscapes. **A** Political positions of representatives in the Swiss parliament 2003, derived from roll call data. **B** Self-reported political position of Swiss citizen grouped by political party they feel close to (data from European Social Survey, ESS 2004). **C** Movie ratings from the Internet Movie Database (imdb.com), including theoretical one parameter fit (from J. Lorenz 2009). **D** Self-reported satisfaction with life from European citizen (data from European Social Survey, ESS 2008).

Fig. 4A shows that distributions of political left—right positions of representatives in the Swiss parliament 2003 derived from roll call data (similar as Leuthold, Hermann, and Fabrikant 2007). Fig.4B shows the histogram of self-reported left—right positions of Swiss citizen from the European Social Survey 2004, colored by their party attachment. Both distributions show a triple-peaked shape with possibly three underlying clusters. Compared to the distribution in the parliament, the distribution of citizens' positions is more narrow, its central cluster is larger, and clusters are less pronounced.

Fig. 4C shows the distribution of movie ratings including a theoretical fit from J. Lorenz (2009), while Fig.e 4D shows the distribution from self-reported subjective quality of life from the European Social Survey 2008. Both distributions are also triple-peaked. They share a small peak at the worst extreme and their majority's peak lies at 8. The movie has an additional peak at the best extreme, while life satisfaction has one additional peak at the very center (as many survey questions). The aim of WP3 is to characterize and describe these nonnormal distributions and to find social or political processes which might generate them.

1.7 Applicant's career and training

I demonstrated my skills in **obtaining formal mathematical results** during diploma and doctoral studies at the University of Bremen (UB) documented by publications on conditions for convergence (J. Lorenz 2005) and convergence to consensus (J. Lorenz and D. A. Lorenz 2010). During my studies and increasingly afterwards as a postdoc at ETH Zurich I performed systematic agent-based **computer simulations of complex socio-economic systems** including density-based formalism of continuous opinion dynamics (J. Lorenz 2006; J. Lorenz 2007; Urbig, J. Lorenz, and H. Herzberg 2008; J. Lorenz 2010; Shin and J. Lorenz 2010), and systemic risk in networks (J. Lorenz and Battiston 2008; J. Lorenz, Battiston, and Schweitzer 2009). I designed, programmed and conducted **decision experiments with humans** on social influence and the wisdom of the crowd (J. Lorenz, Rauhut, Schweitzer, et al. 2011; Rauhut and J. Lorenz 2010) which I continued as the manager of the experimental lab in Oldenburg (J. Lorenz, Rauhut, and Kittel 2013; Tepe, Paetzel, and J. Lorenz 2012). I gathered and analyzed **social and economic data** for J. Lorenz (2009), a paper on the evolution of the productivity distribution in Western economies (König, J. Lorenz, and Zilibotti 2012), and in my current task on measuring social cohesion at Jacobs University Bremen. I learned formal **theories of sociology, economics**

and political science documented by publishing in these disciplines (Groeber, J. Lorenz, and Schweitzer 2013; König, J. Lorenz, and Zilibotti 2012; J. Lorenz 2012b). Many of my publications have a **policy-oriented** spirit: I wrote on designing participatory budgeting (J. Lorenz and Menino 2005), on enforcing and preventing consensus by manipulating communication (J. Lorenz and Urbig 2007), on spurring growth by redistribution (J. Lorenz, Paetzel, and Schweitzer 2013), and on fostering consensus by adding policy dimensions (J. Lorenz 2008), which brought me a prize for applied research in complexity science.

Senior researchers of different disciplines guided me: The mathematician and economist Ulrich Krause advised up to my dissertation, helped by philosopher Rainer Hegselmann; the physicist Frank Schweitzer and his interdisciplinary research group at the Chair of Systems Design at ETH, where I also collaborated with physicist Dirk Helbing; the political scientist Bernhard Kittel at the Center for Social Science Methodology in Oldenburg, where I taught “Social emergence and agent-based models”, “Experimental methods in the social sciences”, and assisted in “Introduction to political science” for social science students. Currently I am guided by social psychologist Klaus Boehnke and sociologist Jan Delhey at Jacobs. I gave a “Language course in mathematics” at the Bremen International Graduate School for Social Sciences (BIGSSS), where I am now affiliated. I am member of the study group Dynamics of Collective Decisions lead by political scientist Susumu Shikano and visited conferences and was invited to colloquiums in mathematics, interdisciplinary physics, sociology and political science.

1.8 Project-Related Publications

Groeber, Patrick, Jan Lorenz, and Frank Schweitzer (2013). Dissonance minimization as a microfoundation of social influence in models of opinion formation. *Journal of Mathematical Sociology*. Accepted.

Lorenz, Jan (2009). Universality of movie rating distributions. *European Physical Journal B* 71, 251–258.

Lorenz, Jan (2010). Heterogeneous bounds of confidence: Meet, discuss and find consensus! *Complexity* 15 (4), 43–52.

Lorenz, Jan, Fabian Paetzel, and Frank Schweitzer (2013). Redistribution spurs growth by using a portfolio effect on risky human capital. *PLoS One* 8 (2), e54904.

Lorenz, Jan, Heiko Rauhut, Frank Schweitzer, and Dirk Helbing (2011). How social influence can undermine the wisdom of crowd effect. *Proceedings of the National Academy of Sciences* 108 (22), 9020–9025.

Lorenz, Jan and Diemo Urbig (2007). About the power to enforce and prevent consensus by manipulating communication rules. *Advances in Complex Systems* 10 (2), 251–269.

2 Objectives and work program

2.1 Anticipated total duration of the project 3 years

2.2 Objectives

The project shall deepen the understanding of opinion dynamics among many actors in continuous policy spaces. This includes an understanding of individuals’ behavior of adjustment with respect to their social and political context as well as an understanding of the emerging system dynamics of opinion landscapes through bargaining, coordination, and information exchange in the light of possible or pending collective decisions.

For a world where technology speeds up deliberation and enables quicker and cheaper voting, the project shall collect evidence if and how methods of direct aggregation of numbers could be democratic innovations for collective decisions about tax rates, budgets and other numerical issues.

This is summarized in the following three objectives:

O1: Description and explanation of human behavior in opinion dynamics and collective decision Understand how humans interact, deliberate and decide in continuous opinion dynamics facing a collective decision about numerical values.

O2: Description and explanation of collective phenomena of opinion dynamics in committees and societies Understand how opinion landscapes evolve in committees and societies as systemic dynamics of continuous opinion dynamics in multi-agent systems.

O3: Procedures for deliberation and decision in continuous policy spaces Define, characterize, and analyze procedures of deliberation and collective decision in continuous policy spaces in the light of real-world applicability for topics as prediction, taxation, and budgeting.

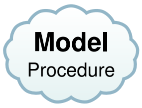
2.3 Work program incl. proposed research methods

The objectives shall be achieved through the tasks in the following research plan with five work packages:

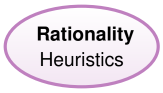
- WP1 Mathematical characterization
- WP2 Experimental analysis
- WP3 Opinion landscapes and their evolution
- WP4 Simulation and systems dynamics
- WP5 Policy proposals for procedures of collective decision

O1 is targeted by doing the tasks in work packages WP1 and WP2, O2 by doing work packages WP3 and WP4. In these work packages models of continuous opinion dynamics based on the modeling framework outlined in Sec. 1.2 will be specified and analyzed starting from different theoretical and empirical perspectives shown in Fig. 5. O3 about possible practical applications is achieved by the tasks in WP5 which accompany the research in the former four work packages on understanding opinion dynamics.

The research concept shown in Fig. 5 integrates six perspectives centered around formal modeling which links the normative perspective to the positivist empirical perspectives as well as the micro- to the macro-level.

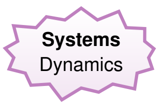


The modeling perspective is to describe mathematically and algorithmically the individual, social, and political process of opinion dynamics among many actors including the procedures of deliberation and collective decision. This includes the definition aggregation procedures, games of opinion formation and collective decisions by actions and payoffs, as well as agent-based stochastic dynamical systems. (Cf. Sec. 1.2)



The rational-choice perspective of decision and game theory is to analyze rational behavior of opinion utterance, belief change, preference change, and individual decision to provide a framework for normative intentions regarding procedures of collective decision or to explain observed behavior. (Cf. Sec. 1.3)

The empirical microlevel is to observe individual behavior in social and political context. In the interplay with modeling this perspective inspires behavioral assumptions in models, validates models by experimental tests, and demonstrates the practical usability of procedures. (Cf. Sec. 1.4)



The systems perspective analyzes static, dynamic and stochastic equilibria of the societal system to explain how it self-organizes the evolution of consensus, bipolarization, fragmentation and extremism. It uses computer simulations, the theory of dynamical systems, and the analogy of human crowds to physical many-particle systems, focusing on phase transitions and patterns of bifurcation. (Cf. Sec. 1.5)

The empirical macrolevel is to observe aggregated data on the committee and society level. In the interplay with modeling it inspires macroscopic descriptions of opinion landscapes and their dynamics which a model shall reproduce and change in desired ways. (Cf. Sec. 1.6)

From a normative social choice perspective we formulate intentions and desired properties a procedure of opinion dynamics and collective decision shall fulfill or foster. For example: truthful utterance, achievement of a (meta)consensus, most correct estimations, increase of welfare, or general acceptance of decisions. (Cf. Sec. 1.1)

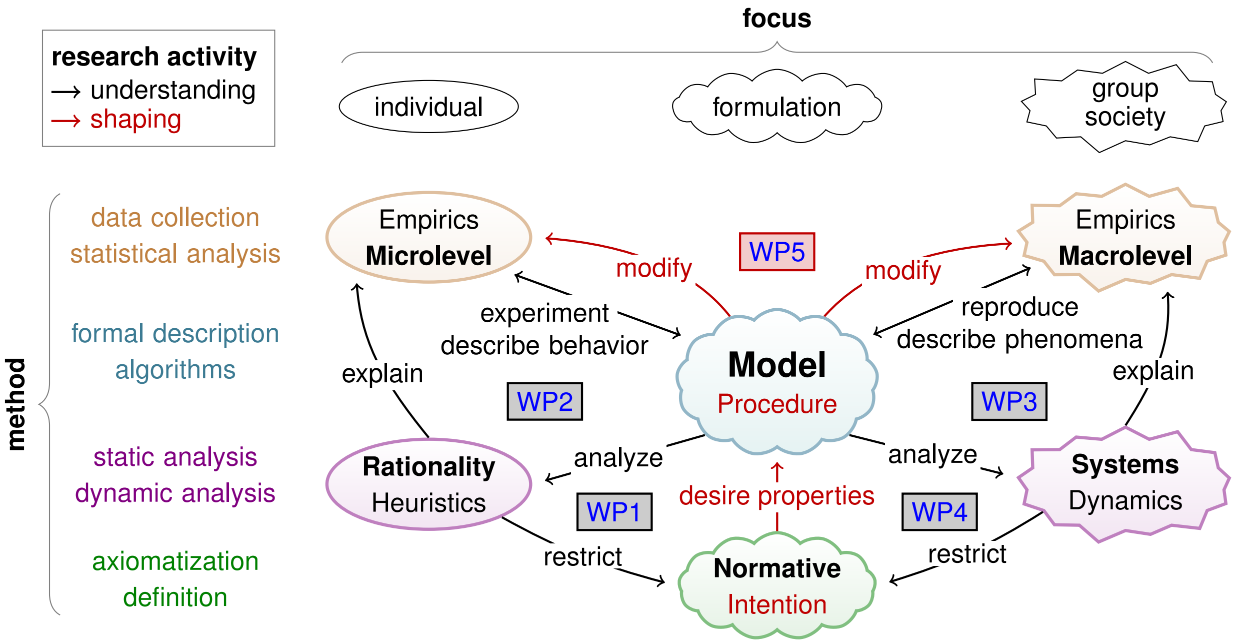
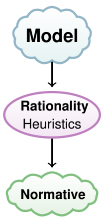


Figure 5: **Research concept** of scientific perspectives and work packages.



Work Package 1: Mathematical characterization

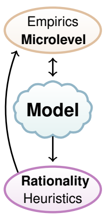
The games outlined in Sec. 1.2 will be mathematically formalized, analyzed and appropriately characterized. This includes the definition of the Bayesian game of opinion formation $\Gamma^{\text{b}} = (N, \mathbb{O}^N, T^N, (p_i)_{i \in N}, (u_i)_{i \in N})$ and the Bayesian game of collective decision $\Gamma^{\text{c}} = (N, \mathbb{O}, T^N, (p_i)_{i \in N}, (u_i)_{i \in N})$, where N is the set of players, T the set of types of players which captures their private information, p_i is the belief of player i about the types of other players and u_i is the utility function of player i mapping the final opinions of all players $(x_1(t), \dots, x_n(t))$, respectively the collective decision $f(x_1(t), \dots, x_n(t))$, to a payoff for player i based on the types of players.

T1 **Analysis: Behavioral rules and utility functions** Assume a given social context of opinions of others and show how social preferences and other parameters as uncertainty and inertia might trigger behavioral heuristics of opinion formation as averaging, balancing, homophily and bounded confidence (cf. Tab. 3 and Tab. 2). In particular, regard situations of non-unique best response and situations where disutility is a function of opinion distance to others as outlined in Groeber, J. Lorenz, and Schweitzer (2013).

T2 **Analysis: Games of opinion formation – Rational equilibrium** Characterize equilibria in games of opinion formation Γ^{b} based on specifications of combinations of individual, epistemic and social preferences (Tab. 2). Characterize critical parameters where the set of solutions structurally changes.

T3 **Analysis: Games of collective decision – Aggregation procedures** Analyze the games of collective decision Γ^{c} produced by the aggregation functions mean, median and mode as listed in Tab. 1. Characterize the mechanisms according to the democratic trilemma framework (see 1.1) and analyse the games for incentive compatibility, strategyproofness, and possible voting strategies.

In terms of game-theoretic branches, T1 is *decision theory* focusing on the individual; T2 is *game theory* looking at all players; and T3 is *mechanism design* taking the social planner's perspective. The tasks deliver hypotheses for experimental tests , well characterized behavioral assumption for agent-based modeling.



Work Package 2: Experimental analysis

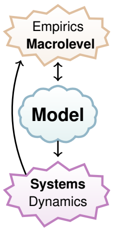
Decision experiments for a controlled computer laboratory environment as motivated in Secs. 1.3 and 1.4 will be designed and conducted. They shall entangle and explore how humans with individual and epistemic preferences and groups decide when they also have social preferences (cf. Tab. 2) or are coupled by aggregation rules of collective decision (cf. Tab. 1). The scope is on middle-sized (5–9) and larger groups (>9). Decisions and utility will be incentivized with money (economic experimental paradigm), but with a sense of a real-world solutions and scalability to the societal level. Specification builds on results of WP1. Further treatment specification regards the communication mechanism.

T1 **Experiments: Games of opinion formation** Design and conduct a series of three experiments based on the **game of opinion formation** as outlined in Sec. 1.2 with combinations of individual, epistemic, and social preferences. The aim is to explore and explain how subjects interact and individually decide when they experience tensions of

- wanting to agree (or disagree) with others, but
- on a value close to the own private ideal point, and
- on a value close to an unknown correct value.

Depart from J. Lorenz, Rauhut, Schweitzer, et al. (2011).

T2 **Experiments: Games of collective decision** Design and conduct a series of three experiments based on the **game of collective decision** with communication as outlined in Sec. 1.2 with combinations of individual and epistemic preferences and different rules of aggregation as listed in Tab. 1. The aim is to explore and explain how subjects interact and groups decide under the three types of rules. Depart from J. Lorenz, Rauhut, and Kittel (2013).



Work Package 3: Opinion landscapes and their evolution

Data of gradual opinions from social surveys and the Internet will be collected. The opinion landscapes are visualized and characterized. Agent-based models are developed to explain their evolution from local interaction, as outlined in Sec. 1.6.

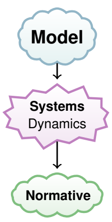
T1 **Data: Collection of individual opinion data** Collect datasets of individual opinions from ordinal or interval scales (e.g. 0,1,...,10; 1,2,3,4,5; or $1\bigstar, \dots, 10\bigstar$). Harvest and compile a dataset with

(a) consumer ratings for the quality of movies (cf. Fig. 4C), games, or general consumer products
(b) political positions, e.g. self-reported positions on the left—right continuum (cf. Fig. 4B)
(c) opinions on values and attitudes from a panel survey, e.g. SOEP
(d) opinions from international representative surveys, e.g. WVS/EVS, EQLS, ESS (cf. Fig.4D)
(e) rate submissions of banks for Euribor or Libor

by using Fig. 1 as model of data storage. Document situation and context of opinion utterance.

T2 **Analysis: Opinion landscapes** Visualize and characterize opinion landscapes based on histograms from data as in Fig. 4 including their evolution over time. Extend to 2d landscapes. Focus characterization of landscapes on measures of the level of cohesion, the fragmentation into clusters, the decay towards the extremes, and anchor effects at boundaries and at the center. Depart from J. Lorenz (2009).

T3 **Models: Evolution of opinion landscapes** Develop agent-based interaction models which reproduce the characterized opinion landscapes. Focus on universal processes which might explain the similarity of the distribution of movie ratings and the subjective quality of life (cf. Fig. 4C, D).



Work Package 4: Simulation and systems dynamics

Agent-based stochastic dynamical models (cf. Fig. 3A) are developed and simulated departing from the outline of Sec. 1.5. The microscopic and structural building blocks to implement are: timing of interaction and communication, functions of adjustment of attitudes and opinions as listed in Tab. 3, rules for utterance of opinions, and rules for influx of new opinions. Focus is put on hidden driving forces to stable or metastable opinion landscapes (cf. Fig. 3B); clustering and other characteristics of such landscapes (cf. WP3); bifurcation of cluster locations at critical parameters (cf. Fig. 3C); and convergence time to and transitions between meta-stable distributions. The phenomena will be related to stylized political phenomena, as the formation of parties or the evolution of cleavages and extremism. An emphasis is put on possible non-trivial interventions to promote certain outcomes.

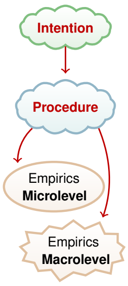
Models are built in NetLogo. The tools for subsequent analysis are dynamical system analysis by quantifying fixed points, systematic computer simulations following the guide in J. Lorenz (2012b), and numerical integration of the corresponding differential and difference equations on the space of agent densities in the opinion space (as in J. Lorenz 2007). Four models are built with different focus of analysis but not necessarily separated.

T1 **Model: Influx of new opinions** Analyze the influx of new opinions with different rates and different distributions departing from Pineda, Toral, and Hernandez-Garcia (2009).

T2 **Model: Smooth bounded confidence** Analyze smoothness of the bounds of confidence and their interplay with the influx of new opinions.

T3 **Model: Multidimensional opinion spaces** Analyze the impact of dimensionality of the opinion space departing from J. Lorenz (2008).

T4 **Model: Heterogeneity** Analyze societies of heterogeneous agents regarding the modes of communication and the adjustment of opinions.



Work Package 5: Policy proposals for procedures of collective decision

Three practical problems of collective decision about numbers are:

- Predicting important quantities, e.g. tax revenue, economic growth, rise of sea level, etc.
- Setting parameters of the schemes of taxation and redistribution such as tax rate, minimal income, etc.
- Fixing the amounts for budget items, e.g. for a number of investment projects.

Alongside the basic scientific work on understanding opinion dynamics in the other work packages, proposals shall be drafted how these collective decision tasks could be set up in the real world and what consequences this might trigger.

T1 **Policy proposal: Collective prediction**

T2 **Policy proposal: Redistributive taxation**

T3 **Policy proposal: Budgeting**

The output of each task is a small policy paper disseminated to the public.

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See also Section 1.8 for references to project-related publications.

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