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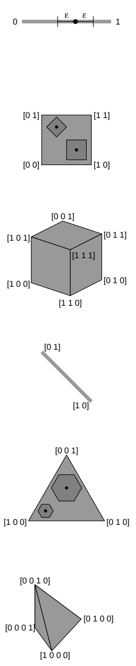
Fostering Consensus

in Multidimensional Continuous Opinion Dynamics under Bounded Confidence

Idea

From social judgement theory and experiments we know that humans tend to agree with others (for normative and informational reasons). But they also tend to ignore others when they have strongly differing opinions.
This basic observation leads to agent-based models of continuous opinion dynamics under bounded confidence, which have analogies to swarm models (like Vicsek's) or communication in distributed computing.
Here, we ask which structural conditions foster the achievement of consensus in the agent's society?

opinion spaces and areas of confidence



Model and Simulation Setup

We consider $n = 200$ **agents**. (Results hold for $n = 50, 500$, too.)
The **opinion space** is a subset of $\mathbb{R}^d$ in which all initial opinions may lie. We distinguish the *cube* $\square^d := [0,1]^d$ and the *simplex* $\triangle^d := \{y \in \mathbb{R}^{d+1}_{\geq 0} \mid \sum_{i=1}^n y_i = 1\}$ with $d = 1, 2, 3$.
We distinguish how agents with opinions $x^1, x^2 \in \mathbb{R}^d$ may measure their opinion distance to others: $\|x^1 - x^2\|_1 = \sum_i |x^1_i - x^2_i|$ and $\|x^1 - x^2\|_\infty = \max_i |x^1_i - x^2_i|$. The **area of confidence** is a region around the agent with the shape of the ball of this p-norms with $p = 1, \infty$ scaled by $\varepsilon > 0$ (the *bound of confidence*).
We distinguish the **communication regimes** of *repeated meetings* (proposed by Hegselmann and Krause) and *gossip* (proposed by Deffuant, Weisbuch and others).

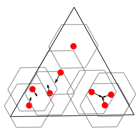
Definition of Opinion Dynamic Processes

Let us consider an initial opinion profile $x(0) \in (\square^d)^n$ or $(\triangle^d)^n$, a bound of confidence $\varepsilon > 0$ and a norm parameter $p \in \{1, \infty\}$.
The **repeated meeting process** $(x(t))_{t\in\mathbb{N}}$ is the time discrete system recursively defined through

$$x(t+1) = A(x(t), \varepsilon)x(t),$$

with $A(x, \varepsilon)$ being the *confidence matrix* defined by

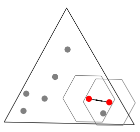
$$a_{ij}(x,\varepsilon) := \begin{cases} \frac{1}{\#I(i,x)} & \text{if } j \in I(i,x) \\ 0 & \text{otherwise,} \end{cases}$$



meeting-step

with $I(i,x) := \{j \mid \|x^i - x^j\|_p \leq \varepsilon\}$. ("#" stands for the number of elements.)
The **gossip process** is a time discrete random process $(x(t))_{t\in\mathbb{N}}$ that chooses in each time step $t \in \mathbb{N}$ two random agents i, j which perform

$$x^i(t+1) = \begin{cases} x^i(t) + \frac{x^j(t) - x^i(t)}{2} & \text{if } \|x^i(t) - x^j(t)\|_p \leq \varepsilon \\ x^i(t) & \text{otherwise.} \end{cases}$$



gossip-step

The same for $x^j(t+1)$ with i and j interchanged.

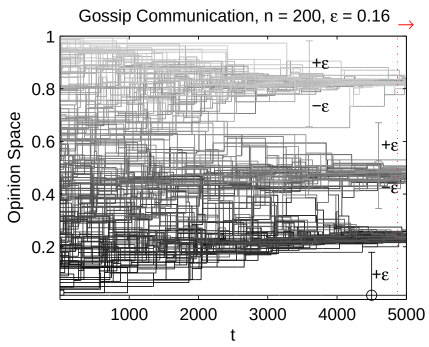
Real-World Interpretations

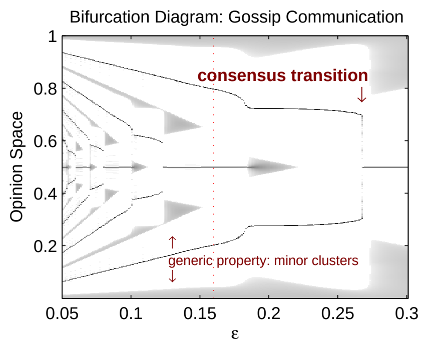
Regarding opinion spaces the simplex stands for issues which can only be changed by changing others in the other direction while the cube represents d issues which can be changed independently. The main example for a simplex opinion is a **budget plan** with a fixed amount of money to allocate.
Agents using the 1-norm are able to **compensate** differences between opinion issues, while agents using the ∞-norm are **non-compensators** and judge each issue independently by ε.
Examples for the group of agents are parliaments, commissions of experts or citizens. Continuous issues can be tax rates, the budget plan, predictions about macroeconomic facts or just attitudes like "I'm more left wing than you.".

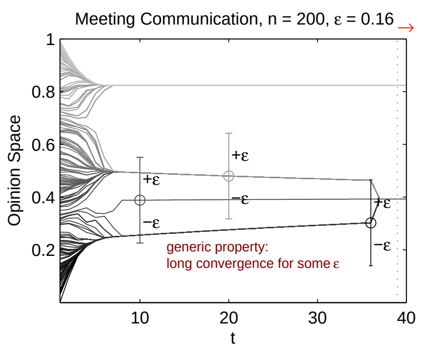
General Dynamics: Clustering and Bifurcations

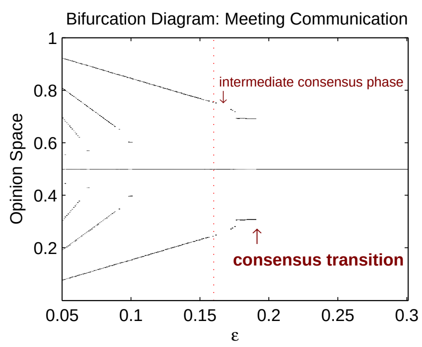
Clustering in time evolution. Every process stabilizes to a clustered configuration. Opinion regions with high density attract other agents.

Bifurcation in ε evolution. There are ε-phases of attractive stabilized profiles with certain numbers, sizes and locations of opinion clusters.



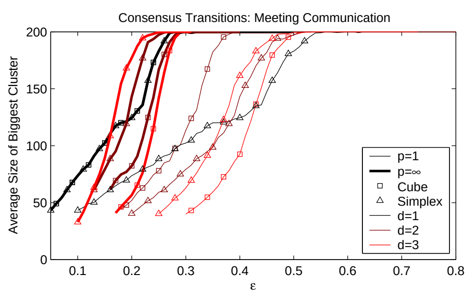


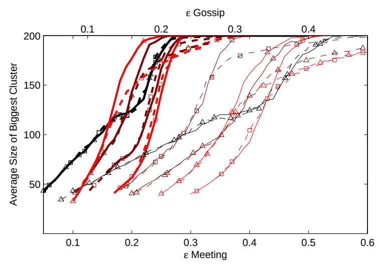




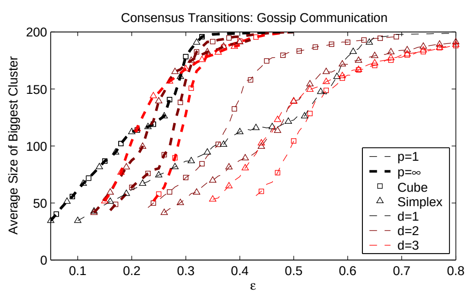
Simulation: Change of the Consensus Transition

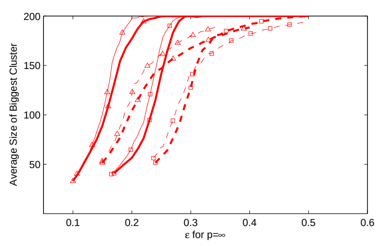
The **average size of the biggest cluster** in the stabilized profiles of 250 simulation runs vs. ε (in steps of 0.01) determines the consensus transition. Simulations for the 24 parameter settings with $\square^d, \triangle^d$, $d = 1, 2, 3$, $p = 1, \infty$ and meeting and gossip.





ε-axis for meeting scaled to 80%.





ε-axis for $p = 1$ scaled to volume equality with $p = \infty$.

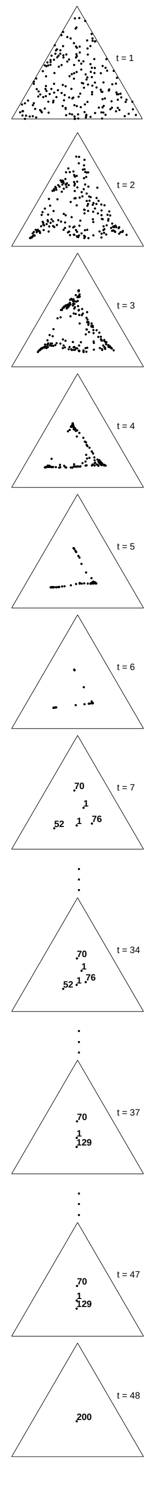
Results about Fostering Consensus

- More issues (rising d) foster consensus if issues are under budget constraints ($\triangle$)! Raising the number of independent issues ($\square$) weakens consensus.
- Switching from gossip to meeting fosters consensus. The consensus transition point is universally 20% lower.
- Compensation (with respect to volume equality of the area of confidence) fosters consensus a little bit.

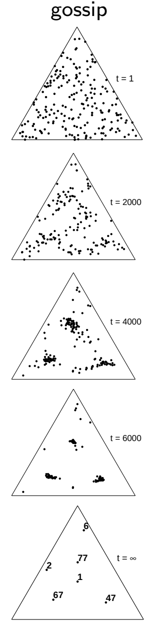
The take-away: If you want consensus, bring more interrelated issues into discussion and initiate big meetings.
If you don't want consensus: Bring more independent issues into discussion and prevent big meetings.

Examples $\triangle^2$
$\varepsilon = 0.2$
$p = \infty$

meetings



gossip



Universität Bremen

The Author holds a Ph.D. Scholarship of FRIEDRICH EBERT STIFTUNG